Math from 1, 2, 3 to XYZ

Welcome!

Please have fun building with your manipulatives!

Math from 1, 2, 3 to XYZ

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Math from 1, 2, 3 to XYZ

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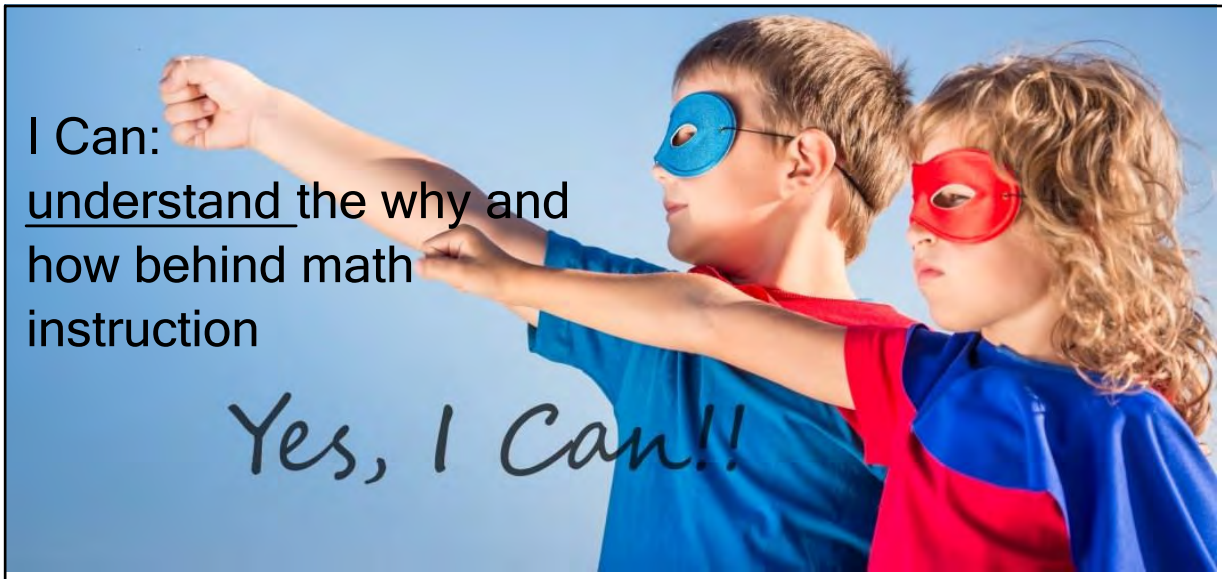
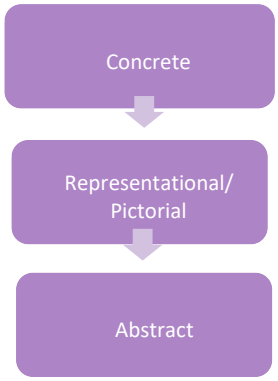
What makes a house structurally strong?



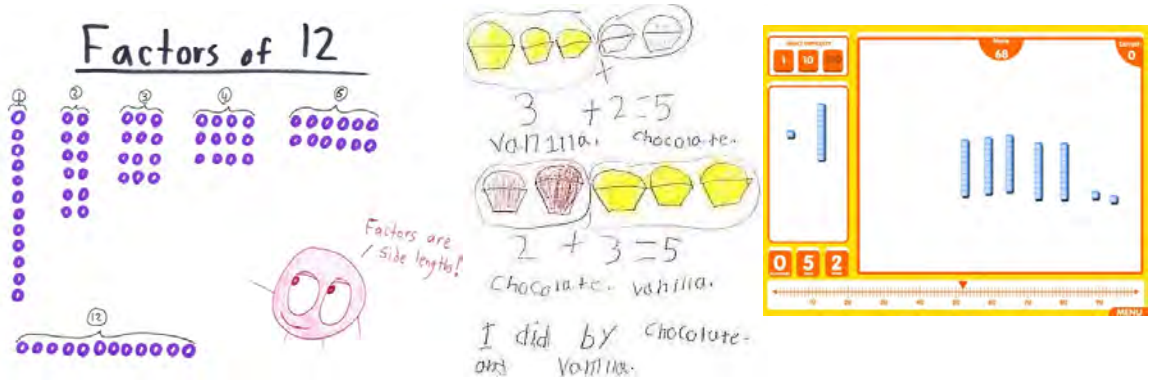
Progression of Learning to Develop Deep Conceptual Understanding in Math



Research has shown that the optimal presentation sequence to teach new mathematical content is through the concrete-pictorial-abstract (CPA) approach (Sousa, 2008).

Concrete

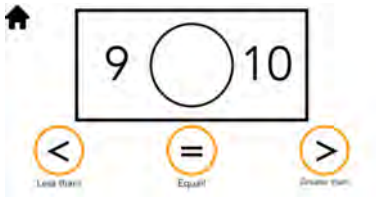
Representational/Pictorial



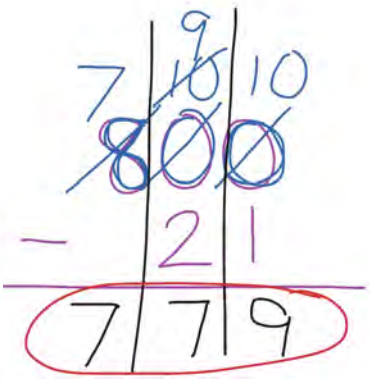
Abstract

15
x 8
80 10 x 8
40 5 x 8
120

(10 x 8) + (5 x 8) = 120

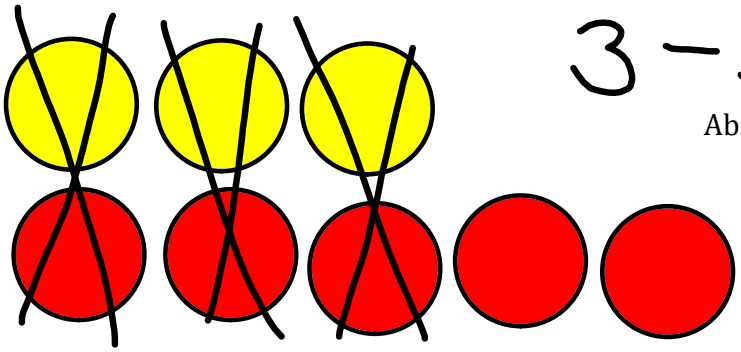


6+4=10
4+6=10



Subtraction Progression

Concrete



Representational

3−5=−2

Abstract

Multiplication Progression

"Groups Of"
Gr. 3 - Equal Size Groups

"Rows Of"
Gr. 3 - Arrays

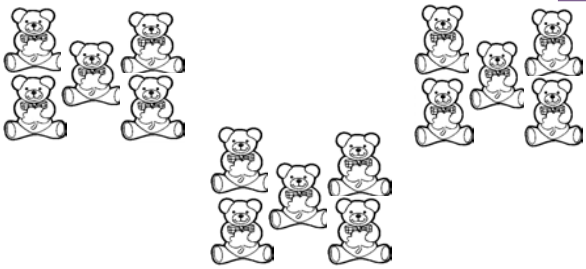
Area Model Strategy

"…Operations and Algebraic Thinking are about understanding and using numbers… It begins with early counting and telling how many in one group of objects. Addition, subtraction, multiplication, and division grow from these early roots."

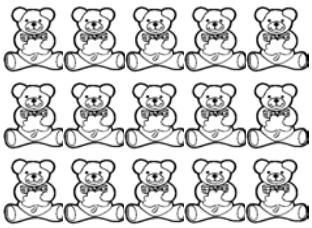
Progressions for the Common Core State Standards in Mathematics
K, Counting and Cardinality; K-5, Operations and Algebraic Thinking (p. 2)

Early Multiplication 3×5

"Equal Size Groups"



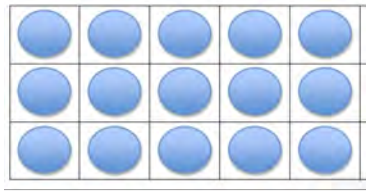
"Arrays"



Understanding How to READ Mathematics Appropriately

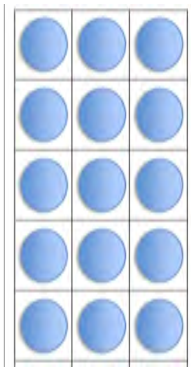
3×5

"3 rows of 5"



5×3

"5 rows of 3"



Understanding How to READ Mathematics Appropriately

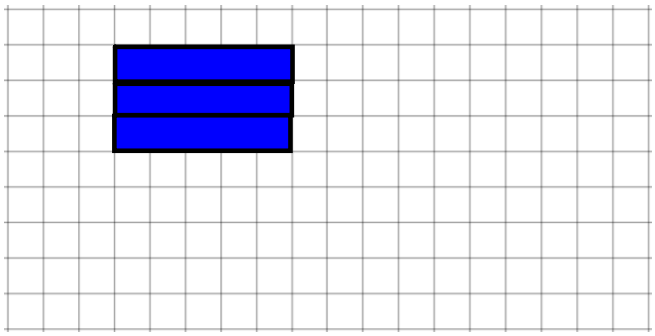
3×5

Same *value* ($15)

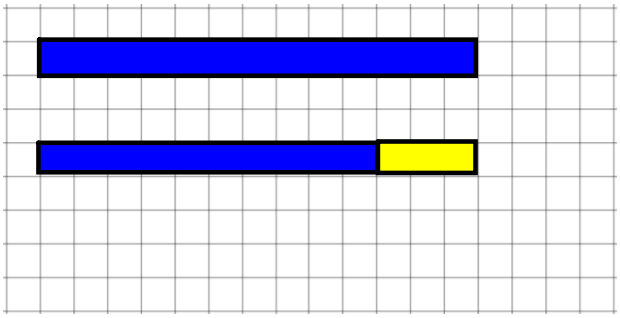
but different *meaning*

5×3

Transitioning from Arrays to the Area Models
3 x 5



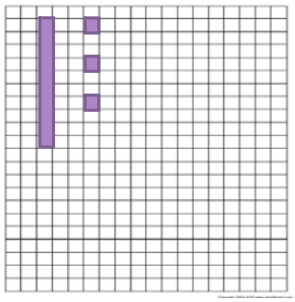
How can you model 13?



Awkward!

10, plus 3 more
Much more manageable!

C.R.A. Trajectory of Learning

Concrete	Representational	Abstract
Base Ten Blocks	*Grid Paper*	*Area Model*
 This is the "doing" phase of learning when student manipulate concrete objects to help them make sense of the mathematics.		13

Multiplying two digits by two digits

- Concrete
- Pictorial/ Representational
- Abstract

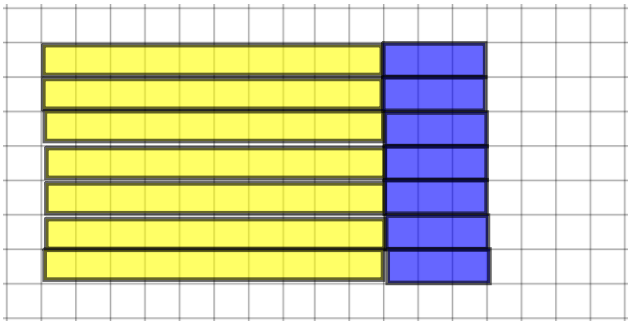
After students have used the tiles (concrete) – they are now ready to move to the pictorial and are able to DRAW out the model to represent the problem

How would you model 7 x 13?
(7 rows of 13)

7 rows of (10 plus 3)

7 rows of 10:

70



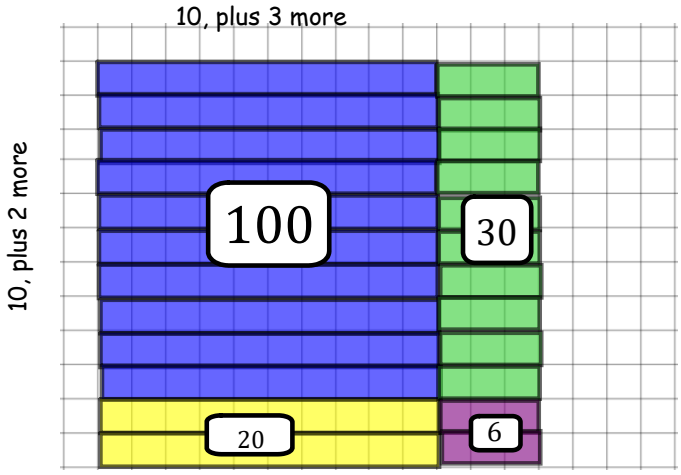
7 rows of 3:

21

Total: 70 + 21 = 91

12 x 13

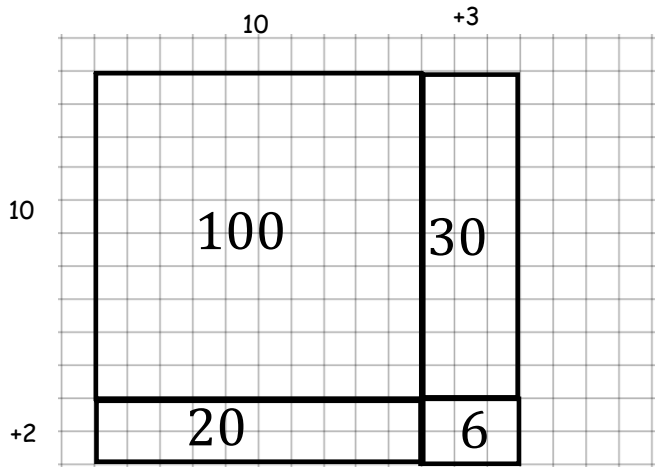
10, plus 3 more



100+30+20+6

= 156

12 x 13



A little more abstract…

Still 100 + 30 + 20 + 6

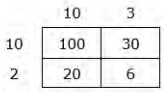
=156

12 x 13

Even more abstract

	10	3
10	100	30
2	20	6

C.R.A. Trajectory of Learning

Concrete	Representational	Abstract
Base Ten Blocks	*Grid Paper*	*Area Model*
		 
This is the "doing" phase of learning when student manipulate concrete objects to help them make sense of the mathematics.		
12 x 13		12 x 13 = 156

46 x 52

	50	2
40	2000	80
6	300	12

2000 +
300 +
80 +
12 =

2392

23 x 15

	20	3
10	200	30
5	100	15

200 +
100 +
30 +
15 =

345

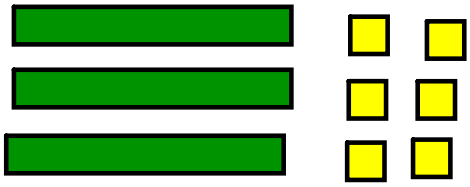
Moving on to Algebra Tiles
Based on understanding of **Area** of Rectangles

1

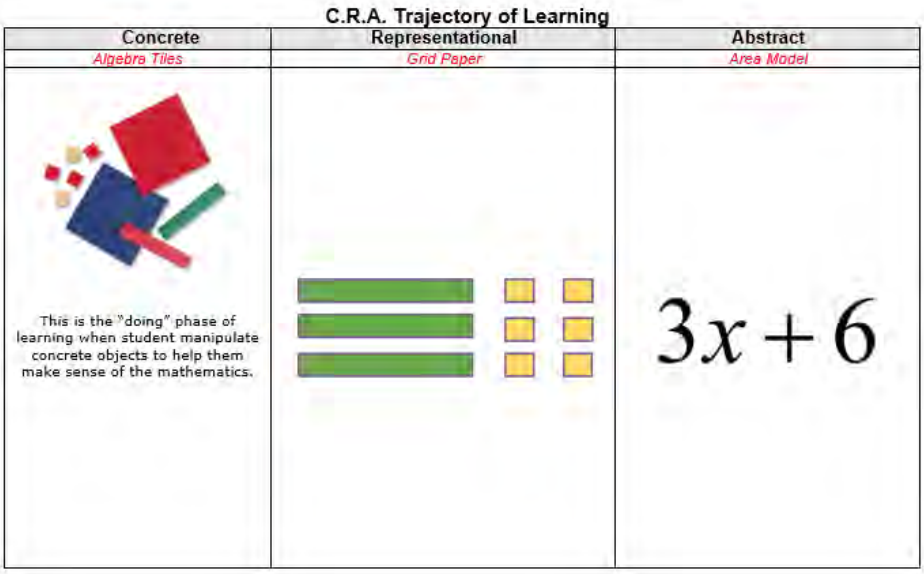
x

Algebra Tiles Extend the Understanding of Area to simplify algebraic expressions

What does this represent?

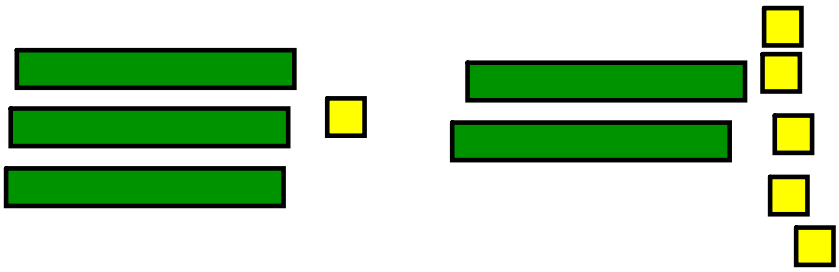


$3x + 6$



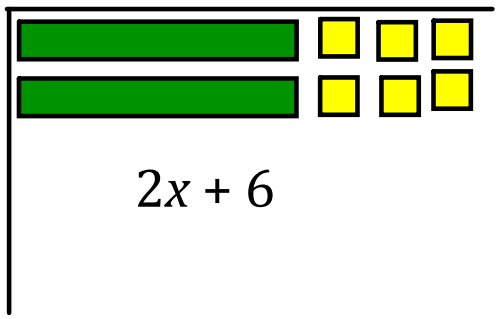
Adding Algebraic Expressions: $(3x + 1)+(2x + 5)$

$5x + 6$



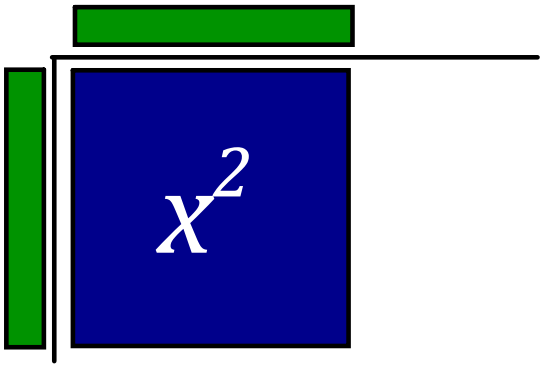
Multiplying Expressions: $2(x + 3)$

"2 groups of $(x + 3)$"



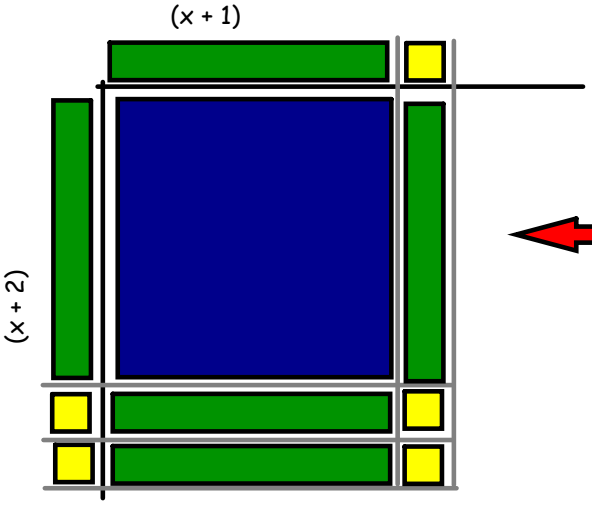
$2x + 6$

What is $x \times x$?



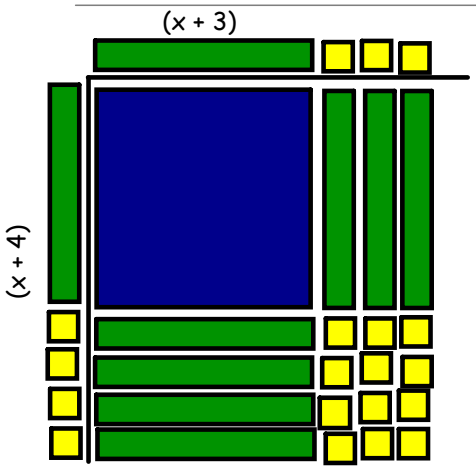
Multiplying Binomials:

$(x + 2)(x + 1)$



$x^2 + 3x + 2$

Multiplying Binomials: $(x + 3)(x + 4)$



$x^2 + 7x + 12$

(x + 3)(x + 4)

$x^2+7x+12$

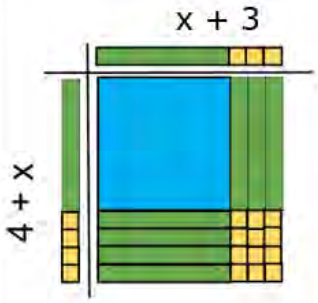
	x	4
x	x^2	4x
3	3x	12

C.R.A. Trajectory of Learning

Concrete	Representational	Abstract
Algebra Tiles	*Grid Paper*	*Area Model*

This is the "doing" phase of learning when student manipulate concrete objects to help them make sense of the mathematics.

$(x + 3)(x + 4)$

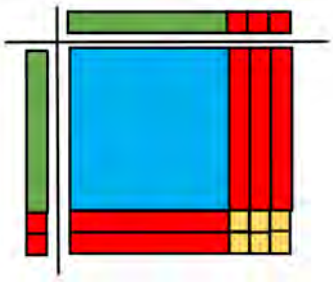


	x	3
x	x^2	3x
4	4x	12

x^2
$3x$
$+ \quad 4x$
12
$x^2 + 7x + 12$

$x^2 + 7x + 12$

Concrete Representational Abstract



	x	-3
x	x^2	$-3x$
-2	$-2x$	$+6$

In this Presentation… Did you???

develop an understanding of the C-P-A (concrete - pictorial - abstract) approach
learn how we've built consistency in the vertical (K-12) curriculum
explore several different mathematical concepts

Thank you!

It was a pleasure presenting for you!

John